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Tugas PDP (F)

1). $x \frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{\lambda y}{x} = 0$, dengan nilai batas $y(0) = 0$; $y'(\pi) = 0$

Bentuk Umum Masalah Sturm-Liouville :

$$(p(x)y')' + q(x)y + \lambda r(x)y = 0$$

Dari soal diperoleh : $p(x) = \frac{1}{2}x^2$ $r(x) = \frac{1}{x}$
 $q(x) = x$

misal : penyelesaiannya $y = x^r$ $y'' = r(r-1)x^{r-2}$
 $y' = rx^{r-1}$

► Substitusi ke persamaan

$$\Rightarrow x \cdot [(r-1)r x^{r-2}] + rx^{r-1} + \lambda \frac{1}{x} x^r = 0$$

$$(r-1)r x^{r-1} + rx^{r-1} + \lambda x^{r-1} = 0$$

$$x^{r-1} [(r-1)r + r] + \lambda x^{r-1} = 0$$

$$x^{r-1} \cdot [r(r-1+1)] + \lambda x^{r-1} = 0$$

$$x^{r-1} \cdot r^2 + \lambda x^{r-1} = 0$$

$$x^{r-1} [r^2 + \lambda x^2] = 0$$

$$\Rightarrow r^2 + \lambda = 0$$

$$r^2 = -\lambda$$

$$r = \pm \sqrt{\lambda} \text{ ;}$$

Sehingga, diperoleh :

$$y = A \cos \sqrt{\lambda} x + B \sin \sqrt{\lambda} x$$

Substitusikan nilai batas :

$$y(0) = A \cos(0) + B \sin(0) = 0$$

$$A = 0, B \neq 0$$

$$y'(x) = -A\sqrt{\lambda} \sin \sqrt{\lambda} x + B\sqrt{\lambda} \cos \sqrt{\lambda} x$$

$$y'(\pi) = 0 + B\sqrt{\lambda} \cos \sqrt{\lambda} x = 0$$

$$\cos \sqrt{\lambda} x = \cos \left(\frac{2n-1}{2} \right) x$$

$$* \sqrt{\lambda} = \frac{2n-1}{2}$$

Jawab : $y_n(x) = B \cdot \sin \left(\frac{2n-1}{2} \right) x$

$$2). y'' + \lambda y = 0, \quad y(0) = 0, \quad y(\pi) = 0.$$

$$\text{misal : } y = e^{mx}$$

$$y'' = m^2 e^{mx}$$

$$\Rightarrow m^2 e^{mx} + \lambda e^{mx} = 0$$

$$(m^2 + \lambda) e^{mx} = 0$$

$$m^2 = -\lambda$$

$$m = \pm i\sqrt{\lambda}$$

$$y(x) = A \sin \sqrt{\lambda} x + B \cos \sqrt{\lambda} x$$

$$y(0) = 0 + B = 0$$

$$B = 0 \rightarrow A \neq 0$$

$$y(\pi) = A \sin \sqrt{\lambda} \pi + 0 = 0$$

$$A \sin \sqrt{\lambda} \pi = 0$$

$$\sin \sqrt{\lambda} \pi = \sin n \pi$$

$$\sqrt{\lambda} = n$$

$$\lambda = n^2$$

$$y = A \sin \sqrt{\lambda} x$$

$$= A \sin nx$$

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